

## S-MATRIX, FEYNMAN ZIGZAG AND EINSTEIN CORRELATION

O. COSTA DE BEAUREGARD

*Institut Henri Poincaré, 75005 Paris, France*

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An inherent binding between Einstein correlations and the  $S$ -matrix formalism entails full relativistic covariance, complete time symmetry, and spacelike connexions *via* Feynman zigzags. The relay is in the past for predictive correlations between future measurements, and in the future for retrodictive correlations between past preparations (Pfleger and Mandel).

An analogy and a partial binding exist between *intrinsic symmetry* together with *factlike asymmetry* of (1) "blind statistical" prediction and retrodiction (retarded and advanced waves, information as cognizance and as will) and (2) positive and negative frequencies (particles and antiparticles). As advanced waves are required for completeness of expansions, "antiphysics" obeying blind statistical retrodiction should show up in appropriate contexts, "parapsychology" being submitted as one of them.

To the Einstein [1,2] paradox<sup>\*1</sup> proper (correlation of measurements upon distant systems that have interacted) corresponds a time-reversed Einstein paradox (correlation of distant preparations that will interact), both very well substantiated experimentally [3,4].

As implied in the mathematics, and as now demonstrated, such facts have been dreaded. [5-7] and are still felt [8] as *extremely* paradoxical. To Einstein [5] they meant "telepathy", to Schrödinger [6] "magic", to de Broglie [7] "upsetting our accepted views concerning space and time". Their existence heralds the advent of a new paradigm, that is, the wording and conceiving of a *Weltanschauung* strictly tailored after the mathematics.

What is intended here is:

(1) A concise and "manifestly covariant" formalization of the mathematics. This has not yet been done, but it should be, because, although the paradox can be expressed in non relativistic quantum mechanics [2], it is in relativistic quantum mechanics that its full significance shows up.

(2) The outlining of a *Weltanschauung* tailored strictly after the mathematical symmetries, the grand

<sup>\*1</sup> Paradox, "a surprising but perhaps true statement" (meaning no. 1 in all dictionaries). Copernicus' heliocentrism has been a paradox.

example here being Einstein's interpretation of the group structure of the Lorentz-Poincaré formulas.

Putting things bluntly, the monster awoken as early as 1927 by Einstein [1] is born from the union of *intrinsic mathematical time symmetry* with Born's principle of *adding partial amplitudes* (rather than probabilities). And, as both genitors have a well established "paradoxical" reputation, *what of the offspring?*

1. Concise and manifestly covariant formalization of the Einstein (predictive and retrodictive) correlations. First we need a general formalization of the  $n$ -tuple Einstein correlations<sup>\*2</sup>.

We assume the existence of a state vector expandable in the form

$$|\Phi\rangle = \sum c^{ij\dots} |\phi_i\rangle |\psi_j\rangle \dots, \quad (1)$$

where the  $|\phi\rangle$ 's  $|\psi\rangle$ 's, ..., span disjoint Hilbert spaces, and also of an operator  $M$  that is the direct product of hermitean operators  $m, p, \dots$ , operating respectively on the  $|\phi\rangle$ 's,  $|\psi\rangle$ 's, ....

<sup>\*2</sup> Garuccio and Selleri [9] and Costa de Beauregard [9] have given the formula in the more restricted, diagonal form:  $|\Phi\rangle = \sum c_i |\phi_i\rangle |\psi_i\rangle$ .

The mean value of the magnitude  $M$ :

$$\langle \Phi | M | \Phi \rangle = \sum \sum c^* i^j \dots c^j i^i \dots \langle \phi_i | m | \phi_i \rangle \langle \psi_j | p | \psi_j \rangle \dots, \quad (2)$$

contains a fully diagonal contribution having the form of a classical sum of partial probabilities, *plus* a composite, off diagonal, interference style, contribution, entailing the "paradoxical" Einstein correlation.

By interpreting, with Dirac [10] and Landé [11], any expansion  $|\phi^A\rangle = \sum_j c_j^A |\phi_j\rangle$  in the form  $\langle A | \phi \rangle = \sum_j \langle A | j \rangle \langle j | \phi \rangle$ , we shall show that, in the Schwinger-Feynman interaction picture, the transition amplitude

$$\langle \Psi_1 | \Phi_2 \rangle = \langle \Psi_1 | U | \Phi_1 \rangle = \langle \Psi_2 | U | \Phi_2 \rangle, \quad (3)$$

between an "initial"  $|\Psi_1\rangle \equiv |\Psi_2(\sigma_1)\rangle$  and a "final"  $|\Phi_2\rangle \equiv |\Phi(\sigma_2)\rangle$  state is of the form (1), where  $U$  denotes that specification of the unitary evolution operator leading from  $\sigma_1$  to  $\sigma_2$ .

Introducing a complete set of orthogonal projectors  $|\Theta\rangle \langle \Theta|$  adapted to the problem considered (for example, predictive correlation polarizations [3] or retrodictive correlation occupation numbers [4]) we rewrite the amplitude (3) as

$$\langle \Psi_1 | \Phi_2 \rangle = \sum_{\Theta} \langle \Psi_1 | \Theta \rangle \langle \Theta | \Phi_2 \rangle. \quad (4)$$

In a predictive problem we interpret [10,11] the  $\langle \Theta | \Phi_2 \rangle$ 's as the components of the final state and the  $\langle \Psi_1 | \Theta \rangle$ 's as the coefficients of the expansion. In a retrodictive problem we interpret the  $\langle \Theta | \Psi_1 \rangle$ 's as the components of the initial state and the  $\langle \Phi_2 | \Theta \rangle$ 's as the coefficients of the expansion. Both expressions are of the form (1). For example, in quantum electrodynamics, the  $|\phi\rangle$ 's in eq. (1) are the photon  $|A\rangle$ , and the electron  $|\bar{\psi}\rangle$ , and the positron  $|\psi\rangle$ , states [12]. We may now interpret  $\langle \Psi_1 | \Phi_2 \rangle$  like  $\langle \Theta | \Phi_2 \rangle$ , regarding  $\langle \Psi_1$  as a label like we interpret  $\langle \Theta$ .\*

One logically missing link in the Schwinger-Feynman formalism was an explicitly covariant definition of the  $|\phi(\sigma)\rangle$  states used initially and finally, and of their hermitean scalar product, etc., by means of 3-fold  $\sigma$  integrals. This has been given [13].

Summarizing this section: (1) *In relativistic quantum mechanics, the Einstein [1] correlations between "presently" separated systems are tied by Feynman zigzags.* (2) *The relay is in the past for predictive correlations between measurements, and in the future for*

*retrodictive correlations between preparations.* (3) *Full relativistic covariance and intrinsic time symmetry of these is thus made formally obvious* — a point now needing a far from trivial epistemological discussion, as our ways of thinking are so macroscopically prejudiced!

2. *Weltanschauung isomorphic to the mathematics.* The time asymmetry of the quantal measurement is of macroscopic origin<sup>†3</sup> as it implies the idea of a repetition of the process, and, thus, a reference to the statistical frequency approach to probability. In an individual quantal event (such as the reception of one photon in the Pflegon-Mandel experiment) there is, and there can be, no intrinsic time asymmetry; but of course, in this case, what is needed is the Bayesian [15] approach to probability — the one consistently (although implicitly) used in this paper. It is now well known [16] that the macroscopic time asymmetry (be it expressed as "blind retrodiction forbidden" [17], or "increasing probability" (Second Law), or "wave retardation") has a "factlike, not lawlike character" [18]. Therefore, if by definition (macro)physics obeys the usual irreversibility statements, and (macro)antiphysics the reversed statements, *microphysics is just as neutral between physics and antiphysics as it is between particles and antiparticles.* There is an analogy between the intrinsic symmetry versus factlike asymmetry of, on the one hand retarded and advanced waves, and of particles and antiparticles on the other hand. Not only is there an analogy, but also a partial binding, through the two expressions of the Jordan-Pauli propagator  $D(x - x')$  as  $(D_{\text{ret}} - D_{\text{adv}})$  and as  $(D_+ + D_-)$ . Not only is there this connection, but the very same argument (completeness for an expansion), entailing the necessary presence of the  $D_+$  and the  $D_-$  contributions in the Fourier expansion of solutions of the covariant wave equation, does entail that of the  $D_{\text{ret}}$  and the  $D_{\text{adv}}$  contributions when solving the covariant position measurement problem [13]. Therefore, by virtue of the very mathematics, it should be expected that, in appropriate contexts, antiphysical evolutions occasionally show up — very much like the positron or the antiproton can be made to show up.

Intrinsic time symmetry in x-space is analogous to intrinsic energy symmetry in k-space.

The intrinsic time symmetry "paradox" is rooted

<sup>†3</sup> Davies [14] makes the point quite clearly.

deeper than in its Loschmidt and Zermelo versions: *inside the probability calculus itself*, because, even if the transition probabilities are symmetric between states (as in card shuffling or in radioactive decay) "blind statistical prediction" is physical while "blind statistical retrodiction" [16] would be antiphysical. Now, Aristotle's concept on the information  $I$  of cybernetics is twofold: *gain in knowledge and organizing power*.

The *learning transition*  $N \rightarrow I_2$  occurs at, say, the reception of a message carrying a negentropy  $N$  and the *organizing transition*  $I_1 \rightarrow N$  at the emission. Notwithstanding the *de facto* inequalities  $I_1 > N > I_2$  (Second Law) there is an *intrinsic symmetry* between the two transitions  $N \rightleftharpoons I$ , and it is in a *one-to-one connection* with the *intrinsic symmetry* between entropy increasing (or physical) and entropy decreasing (or antiphysical) evolutions, and also between "blind statistical" prediction and retrodiction.

Now, the "wavelike probability calculus" (initiated in 1926 by Born in quantum mechanics) brings in a one-to-one binding between retarded waves and blind statistical prediction on the one hand, advanced waves and blind statistical retrodiction on the other — a fact clearly emphasized by Fock [19]. A hermitean scalar product such as  $\langle \Psi_1 | \Phi_2 \rangle$  is symmetric in  $|\Psi_1\rangle$  and  $|\Phi_2\rangle$ , but it can be thought of, and used, asymmetrically, either as the projection of  $|\Psi_1\rangle$  upon  $|\Phi_2\rangle$ , called *collapse* (for *prediction via* retarded waves with sources on  $\sigma_2$ ) or as the projection of  $|\Phi_2\rangle$  upon  $|\Psi_1\rangle$ , which can be called *anticollapse* (for *retrodiction via* advanced waves with sinks on  $\sigma_1$ ).

"Irreversibility of quantal measurements" comes in *via repetition*, that is, with the frequency interpretation of probability. *It then belongs to (macro)physics, and it is factlike, not lawlike*. [18] It comes in *via* von Neumann's ensembles and density matrix. *In fact* von Neumann derives entropy increase from wave retardation (after the time  $t = 0$  of the measurement), while of course entropy decrease would follow from wave advance (before  $t = 0$ ) [20]. This is another wording of Fock's [19] statements<sup>\*4</sup>.

Finally, what would the phenomenology of advanced waves, decreasing probability, blind statistical retrodiction, and information as organizing power,

<sup>\*4</sup> *Factlike* time asymmetry in the  $S$ -matrix formalism is obtained *via* the integration contour in  $k$ -space, by *definition* of the Feynman propagators for virtual particles.

look like? *Exactly* to what parapsychologists call *precognition* and/or *psychokinesis*. *Logically* these phenomena *should* show up, no less than thermodynamical progressing fluctuations — which indeed they are.

Consciousness has two faces symmetric to each other: *cognizance and will*. *Both should show up in the quantal measurement process*.

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